Math Handbook

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Refer to the Math Handbook when you need help with the mathematical concepts that you might encounter in your study of economics.

1.1 Working with Decimals and Percents

Understanding the Skill

A decimal is a number that uses the base-ten value system where a decimal point separates the ones' and tenths' digits. Each place value is ten times the place value to its right. For example, 5.2 is five and two tenths and 12.45 is twelve and forty-five hundredths.

The word *percent* means "per hundred." For example, 5 percent means "5 per 100," or 5/100. If 5 percent of the population is unemployed, then, on average, 5 out of 100 people are unemployed.

To write a decimal as a percent, multiply by 100 percent. To write a percent as a decimal, divide by 100 percent.

Example 1: A company's February sales were 0.125 of its total annual sales. Express this as a percentage.

$0.125 \quad = 0.125 \times 100\%$ *Multiply by 100 percent.*

$= 12.5\%$ *Move the decimal point two places to the right.*

Check your answer: 12.5 should be larger than 0.125 because you multiplied by 100.

Example 2: A company's workforce was 105 percent of what it was a year earlier. Express this as a decimal.

$105\% = \dfrac{105\%}{100\%}$ *Divide by 100 percent.*

$= \dfrac{105}{100}$ *Cancel the % signs.*

$= 1.05$ *To divide by 100, move the decimal point 2 places to the left.*

Check your answer: 1.05 should be smaller than 105 because you divided by 100.

Applying the Skill

1. In 2002, the total personal income in the United States was $8.922 trillion and the total personal taxes were $1.112 trillion. What percent of income were taxes? Round your answer to the nearest hundredth.

2. The GDP of the United States is $11.750 trillion and the GDP of the world is $55.500 trillion. What percent of the world's GDP is from the United States? Round your answer to the nearest tenth.

1.2 Calculating Averages

Understanding the Skill

There are three ways to express the average of a group of numbers. The most common way is to divide the total by the number of values. This kind of average is called the *mean*.

Mean, Median, and Mode Notice that, in Example 1, most workers earn much less than the mean of $27,000; the mean doesn't describe typical earnings well. Typical earnings are often described better by the two other kinds of average: the mode and the median. To determine the mode and the median of a group of numbers, write the numbers in order from smallest to largest. The *mode* is the most common value. The *median* is the middle value. If the number of values is even, the median is the mean of the two middle values.

Example 1: The annual earnings of five workers are shown below. Calculate the mean earnings.

$14,000 $18,000 $14,000 $75,000 $14,000

Solution

Divide the total by the number of values. There are five numbers to average, so the number of values is 5.

$$\text{Mean} = \frac{\text{Total}}{\text{Number of values}}$$

$$= \frac{135,000}{5} \qquad \textit{Simplify the numerator.}$$

$$= \$27,000 \qquad \textit{Divide.}$$

The workers have mean   *Answer the question.*
earnings of $27,000

Example 2: Find the mode and the median of the earnings in Example 1.

Solution

Write the values in order from smallest to largest.

$14,000 $14,000 $14,000 $18,000 $75,000

The most common value is $14,000, so the mode is $14,000.

$14,000 $14,000   **$14,000** $18,000 $75,000

The middle value is also $14,000, so the **median** is $14,000.

Applying the Skill

Calculate the mean, the median, and the mode of each group of numbers.

1. $40,000 $32,000 $38,000 $40,000 $40,000

2. $80,000 $50,000 $35,000 $35,000 $40,000 $60,000

1.3 Calculating and Using Percents

Understanding the Skill

As you recall, the term *percent* means "per hundred." For example, 25% is 25/100. To change a decimal to a percent, move the decimal point two places to the right and add the % symbol.

   0.253 = 25.3%   1.63 = 163%

To calculate and use percents, first write a question. Then rewrite your question as an equation. Replace "percent" with /100, "of" with ×, and "is" with =. Replace the unknown value with a variable, like *x*.

Example 1: Sweden's gross domestic product (GDP) is $255,400,000,000. Agriculture accounts for about $5,108,000,000 of the GDP. What percent of the GDP is from agriculture?

Solution

What **percent of** the GDP **is** from agriculture?	*Write a question.*
What percent of $255,400,000,000 **is** $5,108,000,000?	*Substitute numbers.*
x/**100** × $255,400,000,000 = $5,108,000,000	*Rewrite as an equation.*
$x = \dfrac{\$5,108,000,000}{\$255,400,000,000} \times 100$	*Solve the equation.*
$x = 2$	*Use a calculator.*
Two percent of Sweden's GDP is from agriculture.	*Answer the question.*

Example 2: Sweden's unemployment rate is 5.6% and its labor force is 4.46 million. About how many people are unemployed?

Solution

5.6 **percent of** the labor force **is** how many people?	*Write a question.*
5.6 **percent of** 4.46 million **is how many**?	*Substitute numbers.*
5.6/**100** × 4,460,000 = x	*Rewrite as an equation.*
249,760 = x	*Use a calculator.*
About 250,000 people are unemployed.	*Round your answer.*

Applying the Skill

1. What is 10% of $65?

2. What is 150% of 256,000?

3. $3.75 is 15% of how much?

1.4 Using Ratios

Understanding the Skill

A ratio compares two numbers that have the same units of measure. For example, if Tina runs 8 miles per hour and Maria runs 7 miles per hour, the ratio of their speeds is 8 to 7, or 8:7. A ratio can also be written as a fraction: $\frac{8}{7}$.

You can simplify ratios in the same way that you simplify fractions.

Example 1: Raul runs 8 miles per hour and his little brother Ben runs 4 miles per hour. What is the ratio of Raul's speed to Ben's speed?

Solution

$\dfrac{\text{Raul's speed}}{\text{Ben's speed}} = \dfrac{8}{4}$ *Write a fraction.*

$= \dfrac{8 \div 4}{4 \div 4}$ *To simplify the fraction, divide both the top and the bottom by 4.*

$= \dfrac{2}{1}$ $\frac{2}{1}$ *means the same as* 2:1 *and "2 to 1."*

The ratio of Raul's speed to Ben's speed is 2 to 1.

Ratios are sometimes written as decimals. For example, a company's price-earnings ratio is the ratio of the price of a share of the company's stock to the earnings per share.

Example 2 Suppose a share of a company's stock costs $54.75 and the earnings per share are $2.73. What is the company's price-earnings ratio?

Solution

Price-earnings ratio $= \dfrac{\text{Price of a share of stock}}{\text{Earnings per share}}$ *Write a fraction.*

$= \dfrac{\$55.75}{\$2.73}$ *Substitute numbers.*

$= 20.421245$ *Use a calculator.*

≈ 20.4 *Round your answer.*

The company's price-earnings ratio is about 20.4.   *Answer the question.*

Applying the Skill

1. The GDP of the world is about $55 trillion and the GDP of the United States is about $11 trillion. What is the ratio of the GDP of the world to the GDP of the United States?

2. Suppose a share of a company's stock costs $26.24 and the earnings per share are $3.19. What is the company's price-earnings ratio?

1.5 Calculating Compound Interest

Understanding the Skill

For some savings instruments, interest is calculated and paid multiple times each year. To calculate the amount of each interest payment, you can use the following formula.

$$\text{Interest} = \frac{\text{Balance} \times \text{Interest rate}}{\text{Number of times calculated each year}}$$

If interest on your savings is *compounded,* that means the interest you earn is added to your total savings, and is included in future calculations of interest.

Notice that the interest for the second half of the year is greater than the interest for the first half of the year. In the second half of the year, you earn interest not only on your original balance, but also on the interest you earned in the first half of the year.

Compound interest on loans works the same as compound interest on savings. But for loans, if interest is compounded, you must *pay* interest on interest you haven't yet paid.

Example: If interest on $1,000 is compounded biannually with an interest rate of 5 percent, how much interest do you earn for each compounding period in the first year?

Solution

If interest is compounded *biannually,* that means it is calculated **two** times a year and each compounding period is a half of a year. Calculate the interest for each half.

First half of year $\text{Interest} = \dfrac{\text{Balance} \times \text{Interest rate}}{\text{Number of times calculated each year}}$

$= \dfrac{\$1,000 \times 5\%}{2}$ *Substitute numbers.*

$= \dfrac{\$1,000 \times 0.05}{2}$ *Convert the percent to a decimal.*

$= \$25.00$ *Use a calculator.*

Second half of year

First, calculate the new balance by adding the interest to the old balance.

New balance = Old balance + Interest on old balance

$= \$1,000 + \mathbf{\$25.00}$

$= \mathbf{\$1,025.00}$

$\text{Interest} = \dfrac{\textbf{New balance} \times \text{Interest rate}}{\text{Number of times calculated each year}} = \dfrac{\$1,025.00 + 5\%}{2} \approx \$25.63$

The interest for the first half of the year is $25.00 and for the second half is $25.63.

Applying the Skill

1. If interest on $2,000 is compounded biannually with a rate of 6 percent, what is the interest for each compounding period in the first year? What is the total interest for one year?

2. If interest on $2,000 is compounded four times a year with a rate of 6 percent, what is the interest for each compounding period in the first year? What is the total interest?

3. Look at your answers to Questions 1 and 2. Do you earn more if the interest is compounded twice a year or if it is compounded four times a year?

1.6 Understanding Progressive Taxes

Understanding the Skill

The federal income tax is progressive: a person with a low income is taxed at a lower rate than a person with a higher income. The table at the bottom of the page shows the 2006 income tax brackets for a single person. If you file as single and your taxable income is **$7,125,** you are in the **10 percent** tax bracket because $7,125 is between **$0** and **$7,550.** If your taxable income is **$7,750,** then you are in the 15 percent tax bracket, but your tax is not 15 percent of $7,750. Instead, you pay 10 percent on the first $7,550 of your income. You pay 15 percent on the rest.

Example 1: Calculate tax in the 10 percent tax bracket on $7,125.

Tax = Tax rate x Income in bracket

    = **10% × $7,125**   *Substitute numbers.*

    = 0.10 × $7,125   *Write 10 percent as a decimal.*

    = $712.50   *Multiply.*

The tax on $7,125 is $713.   *Answer the question. Round to the nearest dollar.*

Example 2: Calculate tax in the 15 percent tax bracket on $7,550.

Pay **10 percent** on the first **$7,550.**

    Tax = Tax rate x Income in bracket = **10% × $7,550 = $755.00**

Pay 15 percent on the rest of the income.

    Income in 15% bracket = Total income – **Income in 10% bracket**

    = **$7,750 − $7,550**

    = **$200**

    Tax = Tax rate x Income in bracket = **15% × $200 = $30.00**

Add the tax from the two brackets together to find the total tax.

    $755.00 + $30.00 = $785.00

Applying the Skill

Using the tax bracket table on the right, calculate the taxes on the following taxable incomes:

  A. $7,175   **B.** $17,225
  C. $74,100   **D.** $98,975

2006 Federal Income Tax Brackets (Single)	
Income Bracket	Tax Rate
$0–$7,550	10%
$7,550–$30,650	15%
$30,650–$74,200	25%
$74,200–$154,800	28%

1.7 Creating Line Graphs

Understanding the Skill

A line graph is useful for showing how a value changes over time. You can use a spreadsheet or graphing software to make a line graph. In the example, type the years and inflation rates into the software. The software will do most of the above steps for you. See the software's tutorials or help feature for guidance.

United States Inflation Rate (in percent)									
1970	1971	1972	1973	1974	1975	1976	1977	1978	1979
5.7	4.4	3.2	6.2	11.0	9.1	5.8	6.5	7.6	11.3

Source: U.S. Bureau of Labor Statistics

1. Write the title of the graph and make a grid under it.

2. Write numbers along the left side, or verticle axis, of the grid with 0 at the bottom. The top number should be larger than the largest rate in the table. Label the axis.

3. Write years from the table evenly along the bottom line, or horizontal axis, of the grid. Label the axis.

4. Graph each point where the horizontal line from the **inflation rate** meets the vertical line from the **year.**

5. Draw a straight line to connect each point to the point for the next year.

Example: Make a line graph showing the rate of inflation from 1970 to 1979. Find the largest rate in the table. It is **11.3.**
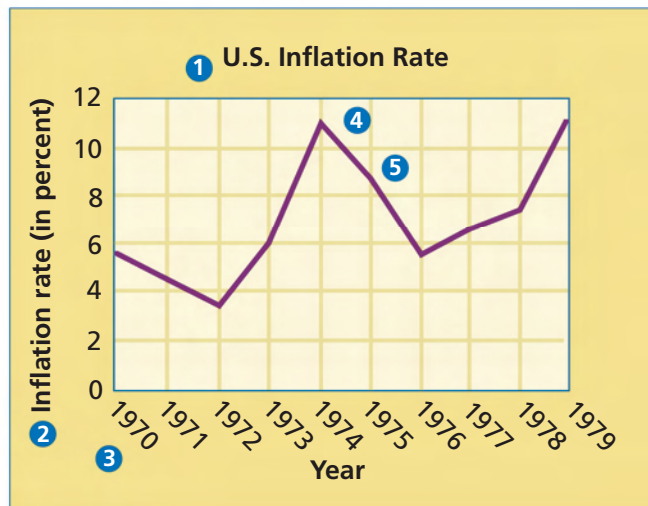
Applying the Skill

Make a line graph to show the unemployment rate from 1993 to 2002.

Unemployment Rate (in percent)									
1993	1994	1995	1996	1997	1998	1999	2000	2001	2002
6.9	6.1	5.6	5.4	4.9	4.5	4.2	4.0	4.7	5.8

Source: U.S. Bureau of Labor Statistics

1.8 Creating Bar Graphs

Understanding the Skill

A bar graph is useful for comparing different values. You can use a spreadsheet or graphing software to make a bar graph. In the example, type the oil consumption information into the software. The software will do most of the above steps for you. See the software's tutorials or help feature for guidance.

| Leading Oil Consumers (in millions of barrels per day)							
United States	European Union	China	Japan	India	Brazil	Russia	Canada
20.0	14.6	6.4	5.6	2.3	2.1	2.8	2.2

Source: CIA World Factbook, 2005 data

1 Write the title of the graph. Draw a box under it.

2 Write numbers evenly along the vertical axis. The largest number should be larger than 20.0 and the smallest number should be 0. Label the axis.

3 Write the names evenly along the horizontal axis. Label the axis.

4 Draw each bar. Find the **oil consumption** on the vertical axis. Imagine a horizontal line from the oil consumption to directly above the **name.** Draw the bar as shown. Each bar should be the same width.

Example: Make a bar graph of the oil consumption of the three greatest oil consumers: the United States, the European Union, and China

Solution

Begin by finding the highest level of oil consumption—the United States with 20 million barrels a day. Then complete the following steps.
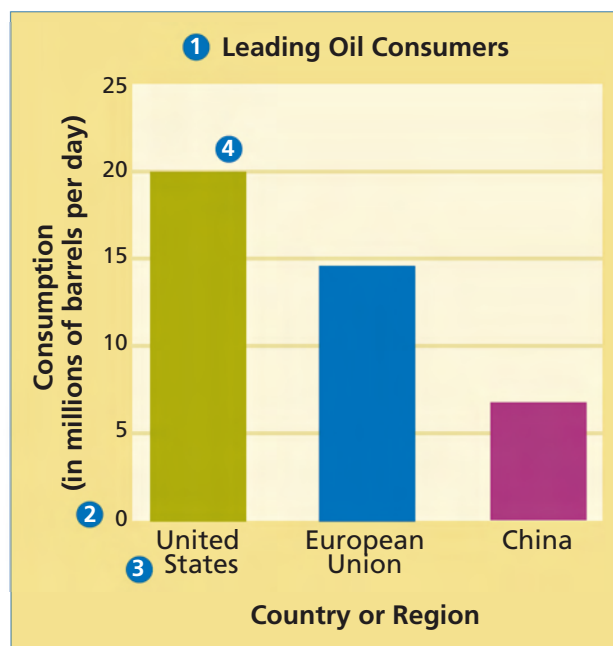
Applying the Skill

Use the information in the table at the top of the page to make a bar graph of the oil consumption of the three leading oil consumers in Asia: China, Japan, and India.

1.9 Creating Pie Graphs

Understanding the Skill

A pie graph is useful for showing the relationship of parts to the whole. The table at the right and the pie graph below show the GDP for various countries and the European Union. The graph makes it easy to see how each GDP contributes to the world's total GDP. The example shows how to make the pie graph.

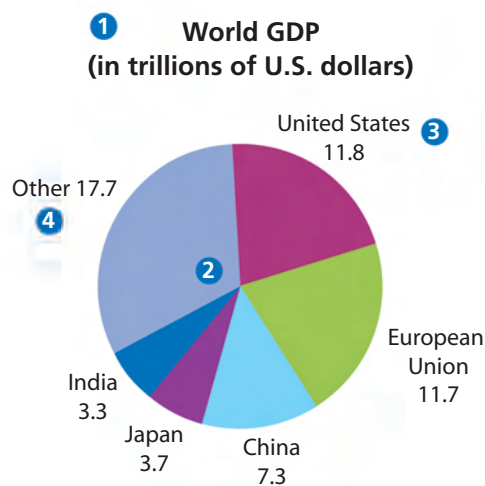| GDP (trillions of U. S. dollars) and Population (millions of people) of Various Countries and the European Union					
Country	GDP	Population	Country	GDP	Population
United States	11.8	296	Russia	1.4	143
European Union	11.7	457	Canada	1.0	33
China	7.3	1,306	Mexico	1.0	106
Japan	3.7	127	South Korea	0.9	49
India	3.3	1,080	Indonesia	0.8	242
Brazil	1.5	186	Other	11.1	2,421

Source: CIA World Factbook, 2005 data

Example

① Write a title for the pie graph.

② Draw a circle. This represents the total GDP of all countries, or $55.5 trillion.

③ Draw a wedge to represent each country or group of countries. Calculate the angle for each wedge using this formula:

$$\text{Angle} = \frac{\text{Country's GDP}}{\text{World GDP}} \times 360°$$

④ Label each wedge with the country name and the amount of GDP. Wedges too small to draw easily can be grouped together and labeled "Other."

① World GDP (in trillions of U.S. dollars)

United States 11.8 **③**

Other 17.7 **④**

②

European Union 11.7

India 3.3

Japan 3.7

China 7.3

Applying the Skill

Use the information in the table at the top of the page to make a pie graph of the populations of the world. Include wedges for the five largest countries, and one for all others.

1.10 Creating a Database

Understanding the Skill

A database is a large collection of information that can be organized and searched. You can use a spreadsheet to make a database. First enter information into the spreadsheet. Label each row and column, including units of measure. You can use the spreadsheet software to manipulate the information and answer questions.

Example Which country has the largest per capita oil consumption?

	A	B	C	D	E	F	G
1		United States	European Union	Japan	Russia	Canada	Mexico
2	Oil Consumption (**millions** of barrels per day)	20.0	14.6	5.6	2.8	2.2	1.8
3	Population (**millions** of people)	296	457	127	143	33	106

Solution

Per capita oil consumption is **oil consumption** divided by **population.** For the U.S. it is

$$\frac{20.0 \text{ million barrels per day}}{296 \text{ million people}} = \frac{20.0 \text{ barrels per day}}{296 \text{ people}} \approx 0.07 \text{ barrels per person per day.}$$

❶ Divide row 2 by row 3. Put the answer in row 4.

❷ Sort the whole database by row 4, from largest to smallest.

	A	B	C	D	E	F	G
1		United States	Canada	Japan	European Union	Russia	Mexico
2	Oil Consumption (**millions** of barrels per day)	20.0	2.2	5.6	14.6	2.8	1.8
3	Population (**millions** of people)	296	33	127	457	143	106
4	Per capita oil consumption (barrels/person/day)	0.07	0.07	0.04	0.03	0.02	0.02

Once the database is sorted, the United States and Canada are the first in the new order. So the United States and Canada are the countries in the database that have the greatest per capita oil consumption, with consumptions of 0.07 barrels per person per day.

Applying the Skill

Make a database of the information in the table on page R10, by entering the information into spreadsheet software. Multiply each GDP by 1 trillion so the units are "dollars" instead of "trillions of dollars." Multiply each population by 1 million so the units are "people" instead of "millions of people".